Local Search

Lecture 6

What search algorithms arise if we **relax our assumptions** – instead of systemically searching alternative paths, evaluate/modify one or more current states?

Agenda

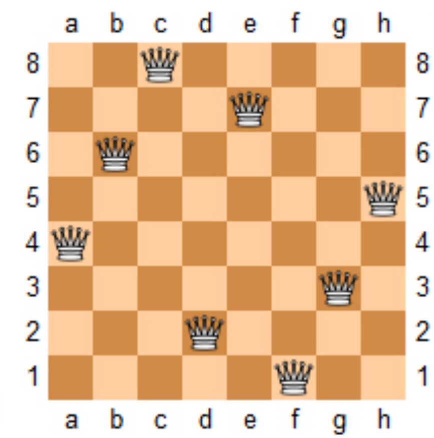
- Local search
  - Optimization
- Objective functions
- Hill Climbing
- Simulated Annealing
- Local Beam Search
- Genetic Algorithms
- Continuous state spaces

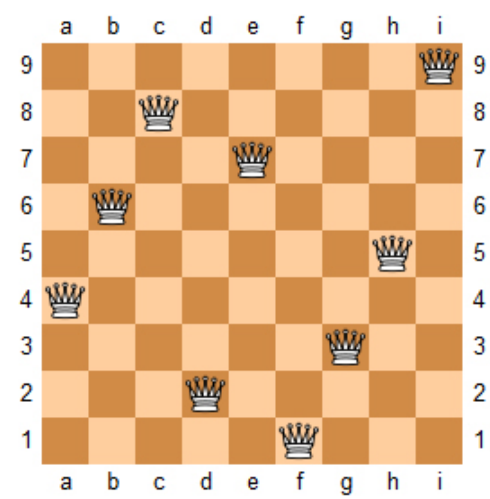
Motivation

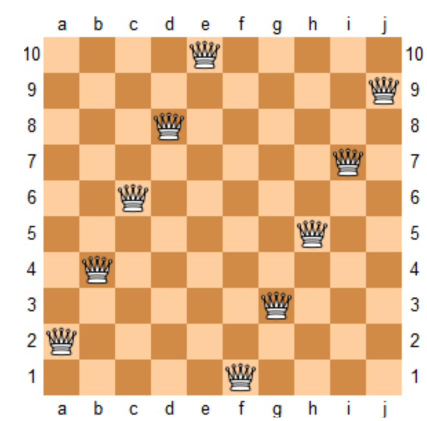
- We have studied algorithms that explore search spaces via paths, keeping alternatives in memory
  - This gets expensive!
- In many problems, path to goal is irrelevant – the final state is all that matters (called **complete-state** formulations, vs. **partial-state**)
- **Local search** algorithms operate using a **current node**, moving only to neighbors, typically…
  - require little memory (often constant)
  - find reasonable solutions (possibly random restarts)

Example: N-Queens

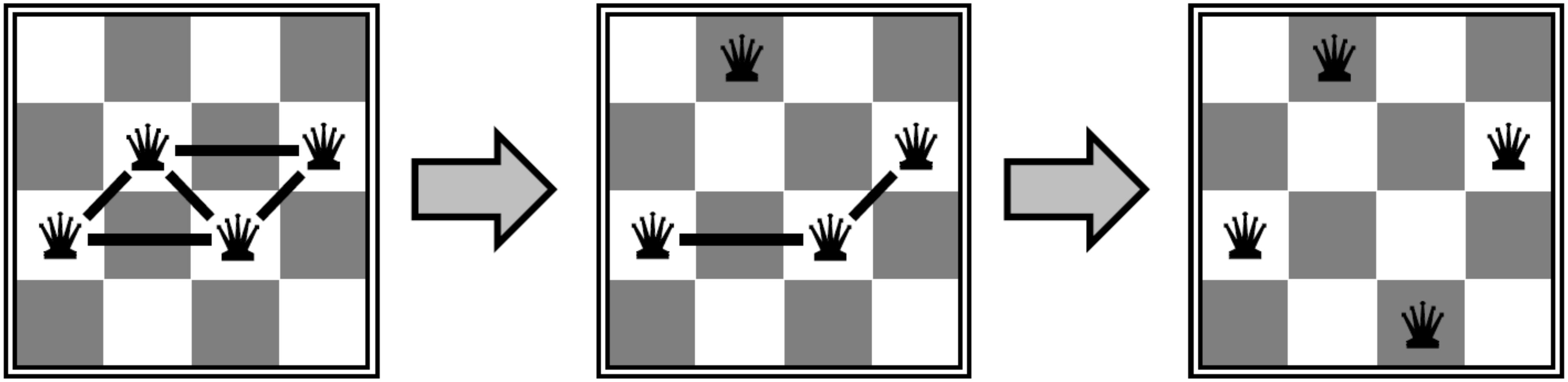
- Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal
  - How many states?
- Notice “path” to solution doesn’t matter, just final placement







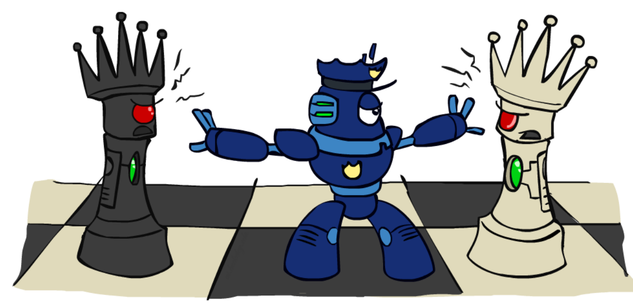
Iterative Improvement



h = 5 h = 2 h = 0

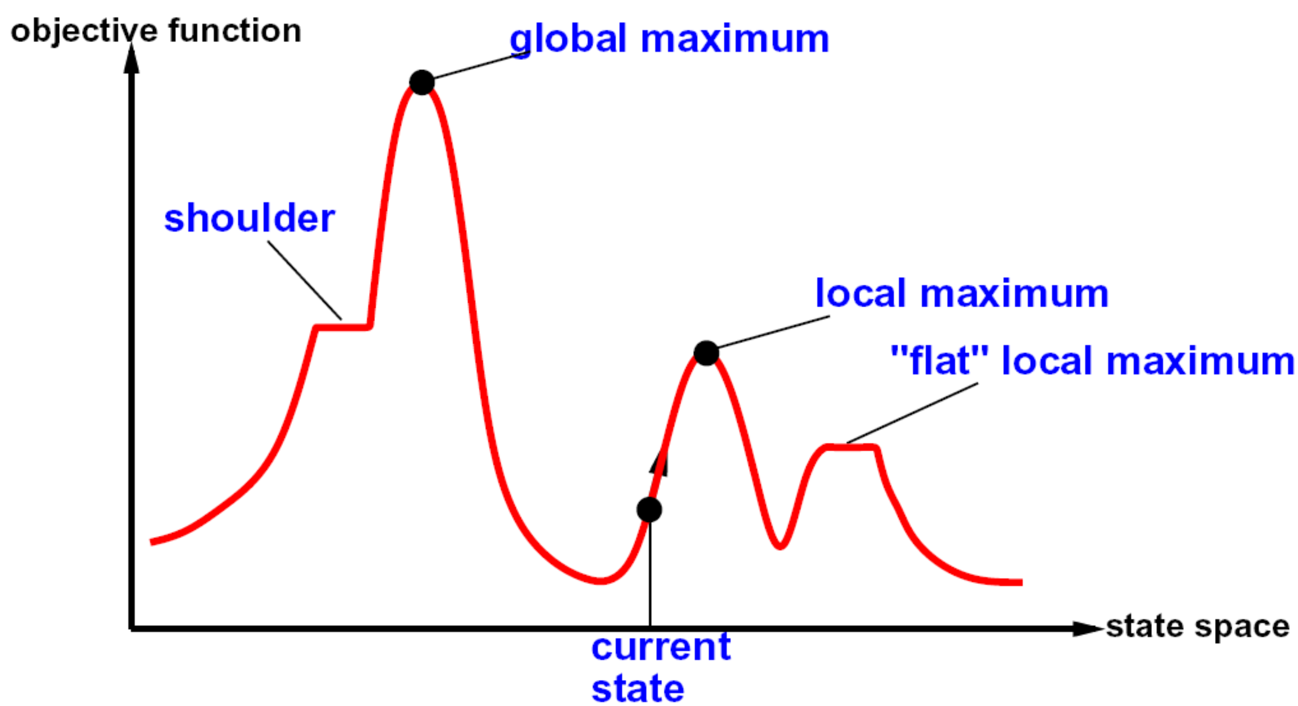
Can typically solve even large problems quickly via incremental moves

N-Queens



Solving Optimization Problems

In addition to search, local search algorithms are useful for solving **optimization problems**, in which the aim is to find the best state according to an **objective function**



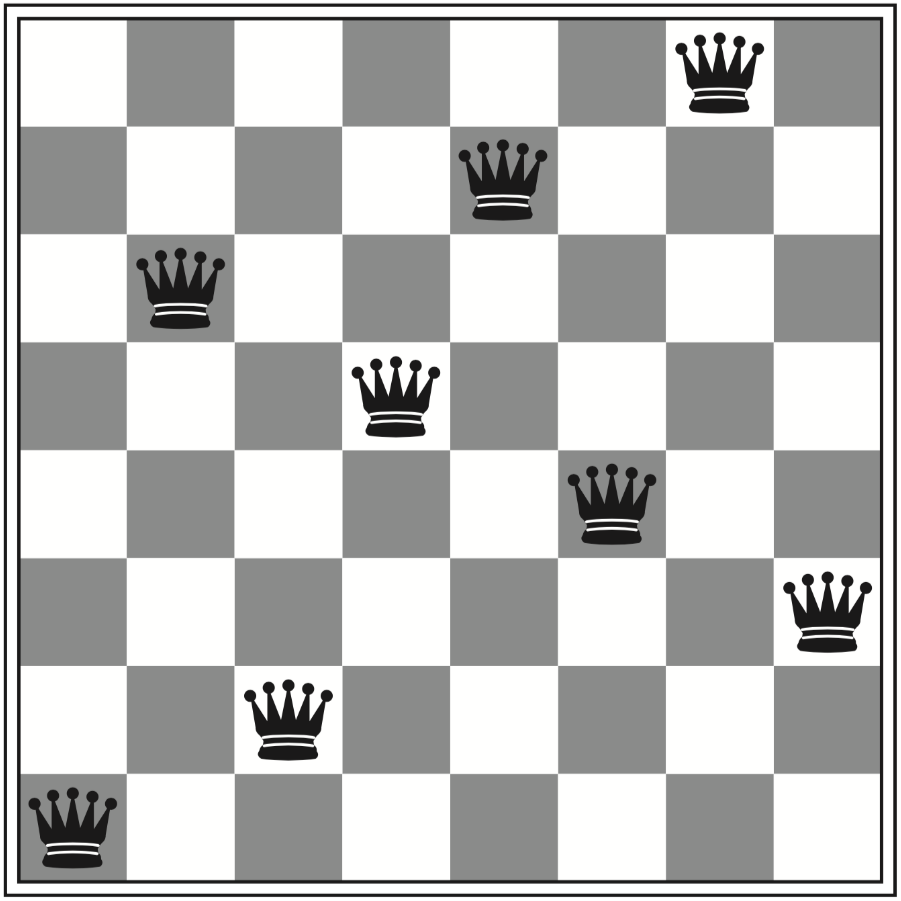
Hill Climbing

Algorithm

- Start wherever
- Repeat: move to the best neighboring state
- If no neighbors better than current, quit

Many variants exist

Checkup

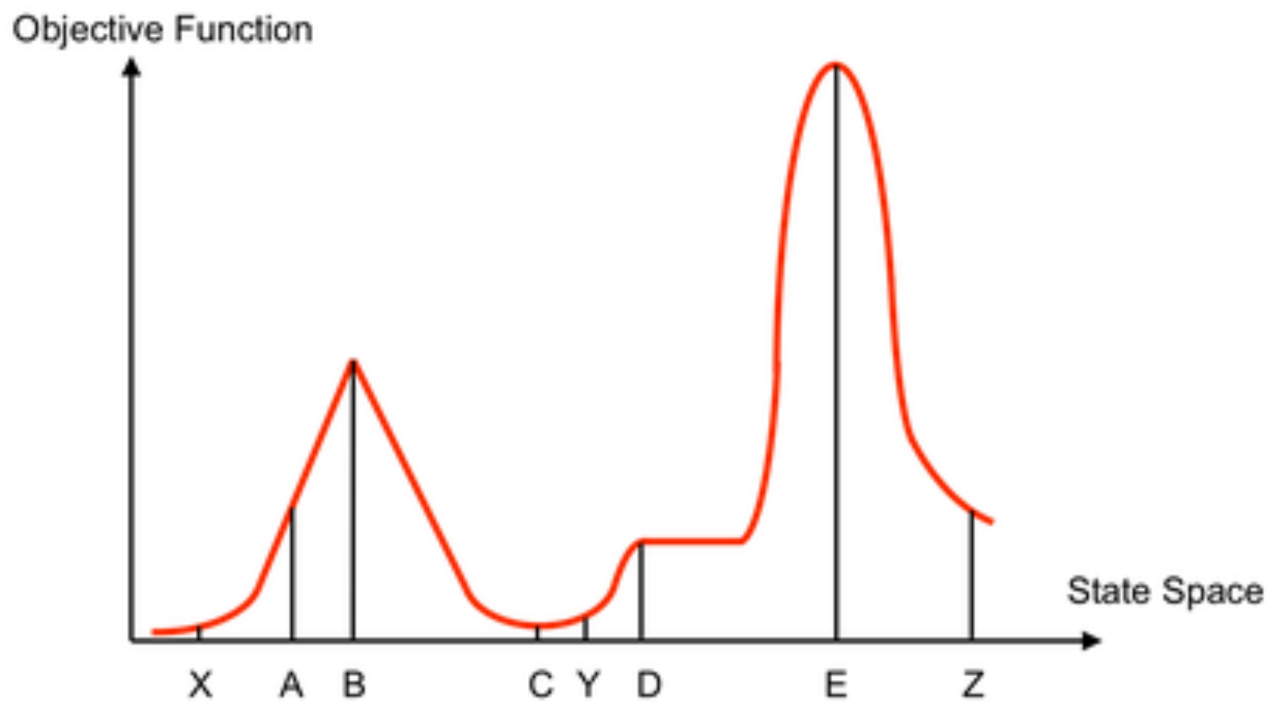


- Complete?
- Random restarts fixes this trivially, why?

Checkup

- Where do you end up if you start from…
  - X
  - Y
  - Z

- Optimal?



Simulated Annealing (1)

- Basic idea: escape local maxima by allowing downhill moves
- But make them rarer as time goes on
- Theory: if slow enough, will converge to optimal state!

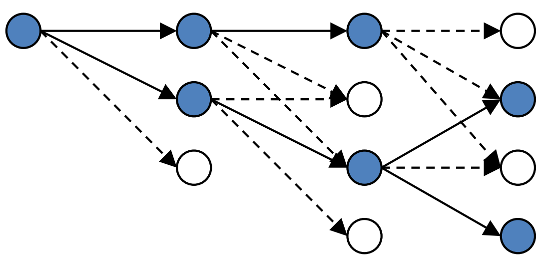
Simulated Annealing (2)

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps

   current ← MAKE-NODE(INITIAL-STATE[problem])
   for t ← 1 to ∞ do
       T ← schedule[t]
       if T = 0 then return current
       next ← a randomly selected successor of current
       ΔE ← VALUE[next] – VALUE[current]
       if ΔE > 0 then current ← next
       else current ← next only with probability e^(ΔE/T)
```

Local Beam Search

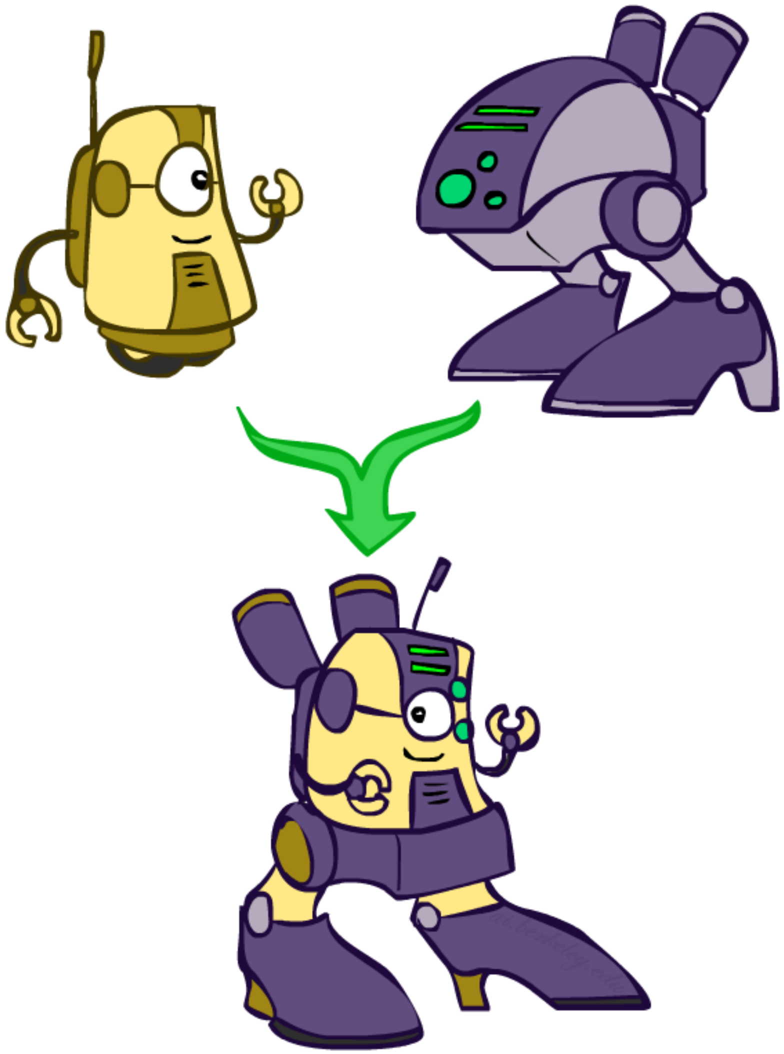
- Start: k randomly generated states
- Loop: generate successors, select k best
  - If any are goal, stop



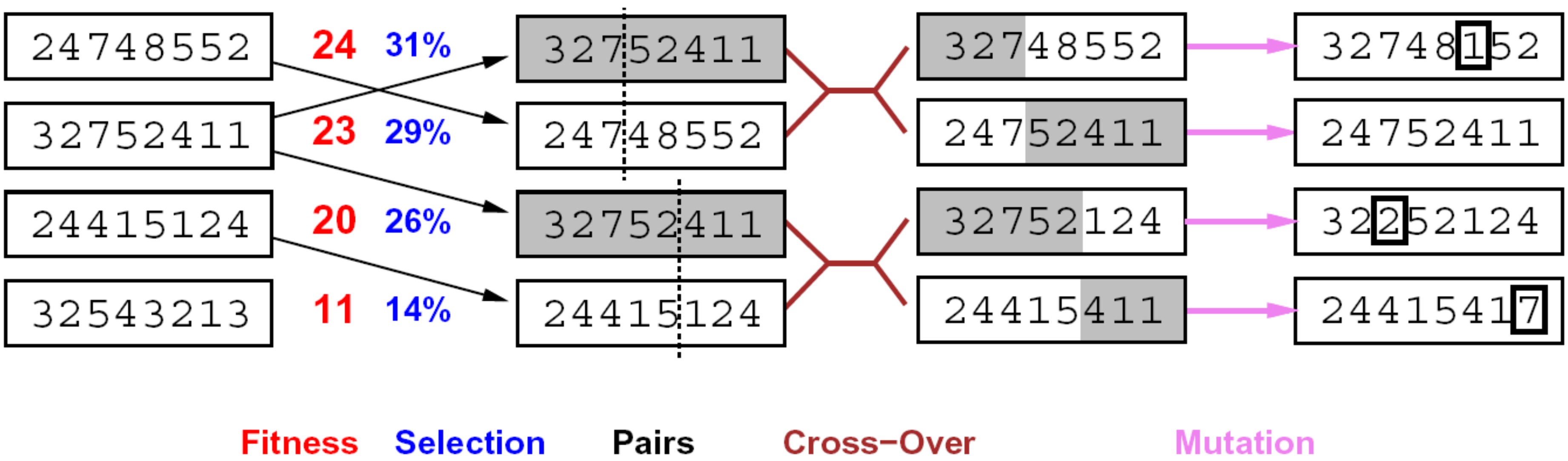
- Variant: **stochastic beam search**
  - Choose k at random, with probability being an increasing function of the value

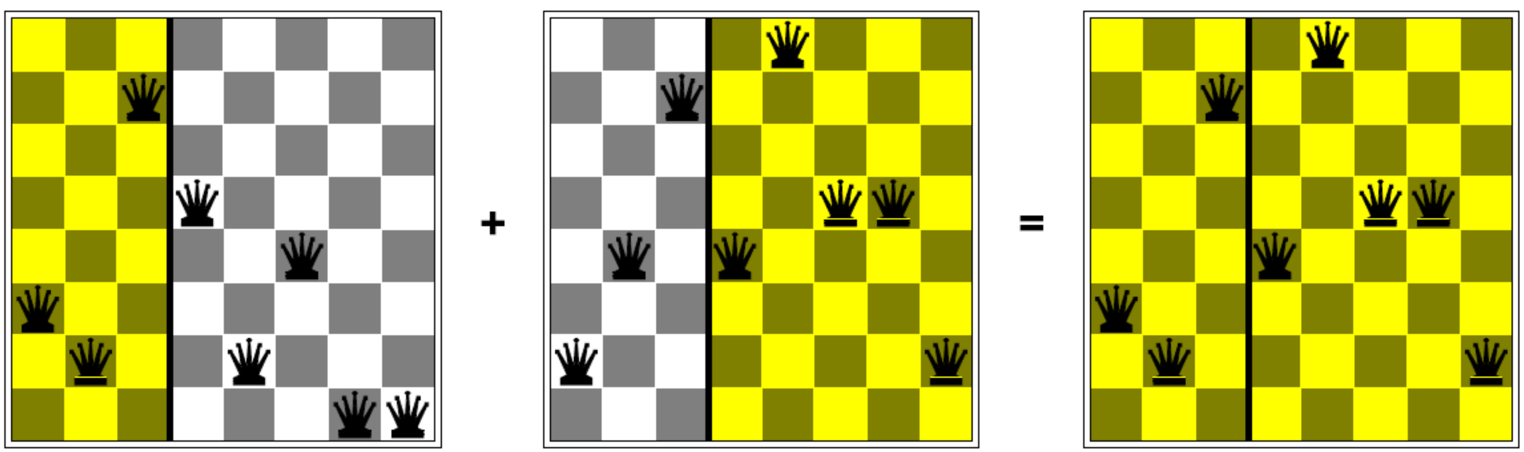
Genetic Algorithms (1)

- Basic idea:
  - stochastic beam search + generate successors from *pairs* of states
- Possibly the most misunderstood, misapplied technique



Genetic Algorithms (2)





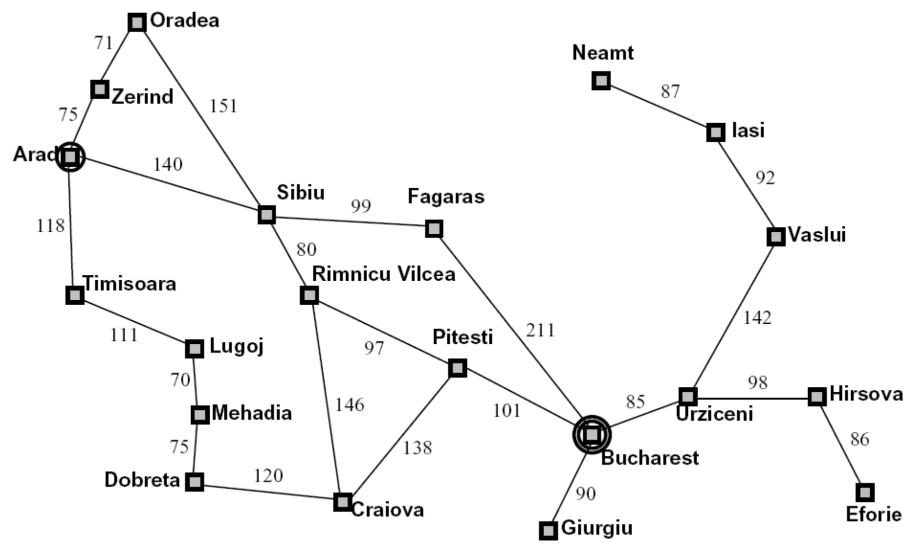
Local Search in Continuous Spaces

- Most of the algorithms we have mentioned thus far work only in discrete state spaces
  – Infinite branching factor!
- Sometimes we can take a continuous problem and discretize the neighborhood of each state
- But how to perform truly continuous search?
  – Long topic, here’s a flavor

Example

- Where to locate three airports in Romania?

- Objective: minimize the sum of squared distances to each city on the map



$$f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C} (x_i - x_c)^2 + (y_i - y_c)^2$$

Using the Gradient

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

$$\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C} (x_i - x_c)$$

- Finding the magnitude and direction of steepest slope could be used to find the minimum via **gradient methods**

- Often cannot be solved in closed form, so the **empirical gradient** is determined via evaluating the response $\pm\delta$

Hill Climbing

$$\boldsymbol{x} \leftarrow \boldsymbol{x} + \alpha \nabla f(\boldsymbol{x})$$

The step size (α) is a small constant

- Variety of methods for choosing/updating the value

Newton's Method

- A method for finding successively better approximations to the roots of a function

  $f(x) = 0$ via $x \leftarrow x - \frac{f(x)}{f'(x)}$

- In this case, finding extrema via…

$$\boldsymbol{x} \leftarrow \boldsymbol{x} - \boldsymbol{H}_f^{-1}(\boldsymbol{x}) \nabla f(\boldsymbol{x})$$

where H is the Hessian

Summary

- **Local search** methods operate on **complete-state** formulations and keep only a small number of nodes in memory
- **Hill-climbing** methods can get stuck in **local optima** and stochastic methods, such as **simulated annealing**, can return optimal solutions under certain conditions
- A **genetic algorithm** is a stochastic hill-climbing search operating over a large population in which new states are generated by mutation and crossover
- Local search in continuous spaces often involves evaluating the **gradient** of the objective function